

Chapter 8

TicTacToe Magic

Three simple games are related in a surprising way.

The first of the three games is Pick15. Harold Stark, who was then at the University of Michigan, told me about the game in the late 1960s. I suspect that this is the first time you have heard of it.

The game involves two players. You start by listing the single digit numbers from 1 to 9. You then take turns selecting numbers from the list, attempting to acquire three numbers that add up to 15. Each number can be chosen only once. You may eventually acquire more than three numbers, but you must use exactly three of them to total 15. If neither player can achieve the desired total, the game is a draw.

For example, suppose that Green and Blue are playing. They start with the list.

List : 1 2 3 4 5 6 7 8 9
Green :
Blue :

Suppose Green has the first move and chooses 8. Then Blue chooses 4 and Green chooses 2. Now Blue should respond by choosing 5 to prevent Green from getting $2 + 5 + 8 = 15$. Here is the situation after the first two rounds.

List : 1 ~~2~~ 3 ~~4~~ ~~5~~ 6 7 ~~8~~ 9
Green : 2 8
Blue : 4 5

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Now Green chooses 6 to block $4 + 5 + 6 = 15$ from Blue. This is actually an advantageous move for Green because it gives her two different ways to win, $1+6+8$ and $2+6+7$. Blue cannot block both. If Blue chooses 7, then Green chooses 1 to win. If Blue chooses 1, then Green chooses 7 to win. The final position might be

```
List : 1 2 3 4 5 6 7 8 9
Green : 2 6 7 8
Blue  : 1 4 5
```

Note that Green also has $7+8 = 15$, but this does not count because there are only two numbers in the sum.

Figure 8.1 shows the starting position for the Pick15 option in our MATLAB `tictactoe` program. When you play against the computer, your moves are shown in green and the responses from the program are shown in blue.

1	2	3	4	5	6	7	8	9
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Figure 8.1. Starting position for Pick15.

Figure 8.2 shows the position for our example game after two moves from each player. Green now has to choose 6 to block Blue.

1	2	3	4	5	6	7	8	9
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Figure 8.2. Position after two moves from each player.

Figure 8.3 shows the final position for our example game. Green has won with $2+6+7 = 15$.

1	2	3	4	5	6	7	8	9
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Figure 8.3. Final position for Pick15.

Please take time out from reading this chapter to try this game a few times yourself, playing against a friend on paper or against our MATLAB program. I think you will find that Pick15 is more challenging than it sounds at first.

The second of our three games is familiar worldwide. It is called “TicTacToe” in the United States, “Noughts and Crosses” in Great Britain, and has many other names in many other countries and languages. Our program `tictactoe` uses green and blue instead of X’s and O’s. The objective, of course, is to get your color on all three squares in a row, column, or diagonal. Figure 8.6 shows typical output.

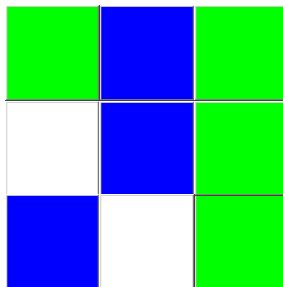


Figure 8.4. Typical output from a game of *TicTacToe*. Green has won with the third column.

Our program `tictactoe` uses a naive three-step strategy.

- If possible, make a winning move.
- If necessary, block a possible winning move by the opponent.
- Otherwise, pick a random empty square.

This strategy will lead to a win only when the opponent makes a error. And, even though it is possible for the player with the second move to always force a draw, this strategy will not accomplish that.

Our third game, *Magic15*, introduces the Lo-Shu 3-by-3 magic square. Now we see that *Pick15* is actually *TicTacToe* played on a magic square. The rows, columns and main diagonals of the magic square provide all possible ways of having three distinct numbers that sum to 15. Winning moves in *Pick15* correspond to winning moves in *TicTacToe*. All three games are actually the same game with different displays.

8	1	6
3	5	7
4	9	2

Figure 8.5. Initial configuration for a game of *Magic3*.

8	1	6
3	5	7
4	9	2

Figure 8.6. Green has won with $6+7+2 = 15$ in the third column.

Exercises

8.1 *Traditional.* Modify `tictactoe.m` so that it uses traditional X's and O's.

8.2 *Win.* Is it possible to win against `tictactoe` with its naive strategy?

8.3 *First move.* Modify `tictactoe` so that the computer takes the first move.

8.4 *Center square.* Modify the strategy used by `tictactoe.m` so that, before taking a random move, it takes the center square if it is available. Does this improve the program's chances of winning or forcing a draw?

8.5 *Computer versus computer.* Modify `tictactoe.m` so that the computer plays against itself. Count the number of draws and wins for both sides. Run a large number of games, with and without the addition of the center square strategy.